Classification and Regression Trees

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Math 243: Stat Learning

November 8th, 2021

Outline

In today's class, we will...

- Discuss decision trees as a non-parametric model
- Investigate pruning algorithms for improving accuracy of trees

Section 1

Decision Trees

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- What makes an effective question?

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  - Yes/No questions represent a branching point or node
  - The final guess represents the prediction
- What makes an effective question?
  - Separates data into roughly equal sizes
  - Data in each group relatively are similar
  - Later questions should be based on answers to earlier questions.
  - Early questions are general, later questions are specific.

My Favorite Book

Previously asked questions:

1. Is it set in the UK? **No**
2. Is it about a sick day? **No**
3. Does it take place on an island? **No**
4. Is it set in California? **No**
5. Are there any gunshots in the book? **No**
6. Is the book less than 200 pages? **No**

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Section 2

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2. The method then repeats step 1 for each of the two groups S_1 and S_2.
3. The method continues splitting groups until each subdivision has few observation (or another predetermined stopping condition is met)

Trees on Trees

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- The data was collected by the Portland Parks and Rec's *Urban Forestry Tree Inventory Project*.
- The Tree Inventory Project has gathered data on Portland trees since 2010, collecting this data in the summer months with a team of over 1,300 volunteers and city employees.

pdxTrees Data

- The `pdxTrees` dataset is too large to install alongside the package. Instead, the package provides helper loading functions:
  - `get_pdxTrees_parks()` pulls data on 25,534 trees from 174 Portland parks
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- To keep things manageable, we'll focus on trees in parks nearby Reed.

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library(pdxTrees)
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                                          "Woodstock Park","Berkeley Park", "Powell Park"))
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dim(my_pdxTrees)
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## [1] 1039   34
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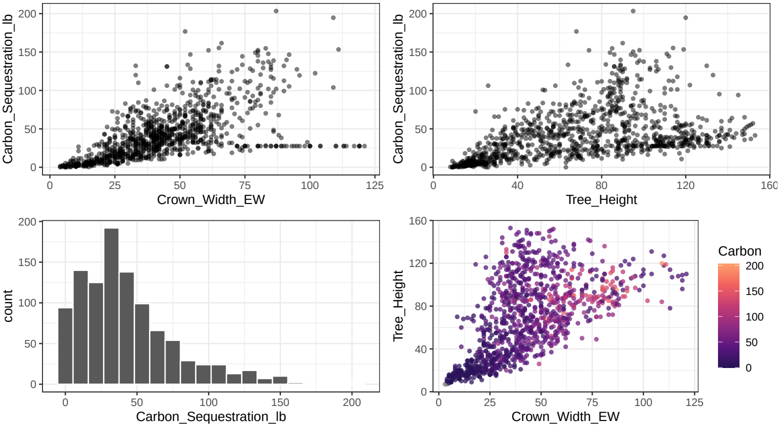
```
##  [1] "Longitude"                 "Latitude"
##  [3] "UserID"                    "Genus"
##  [5] "Family"                    "DBH"
##  [7] "Inventory_Date"            "Species"
##  [9] "Common_Name"               "Condition"
## [11] "Tree_Height"               "Crown_Width_NS"
## [13] "Crown_Width_EW"            "Crown_Base_Height"
## [15] "Collected_By"              "Park"
## [17] "Scientific_Name"           "Functional_Type"
## [19] "Mature_Size"               "Native"
## [21] "Edible"                    "Nuisance"
## [23] "Structural_Value"          "Carbon_Storage_lb"
## [25] "Carbon_Storage_value"      "Carbon_Sequestration_lb"
## [27] "Carbon_Sequestration_value" "Stormwater_ft"
## [29] "Stormwater_value"          "Pollution_Removal_value"
## [31] "Pollution_Removal_oz"      "Total_Annual_Services"
## [33] "Origin"                    "Species_Factoid"
```

Carbon Sequestration

- Can we predict carbon sequestration based on other tree features?



An Old Friend

This seems like a good time to implement linear regression:

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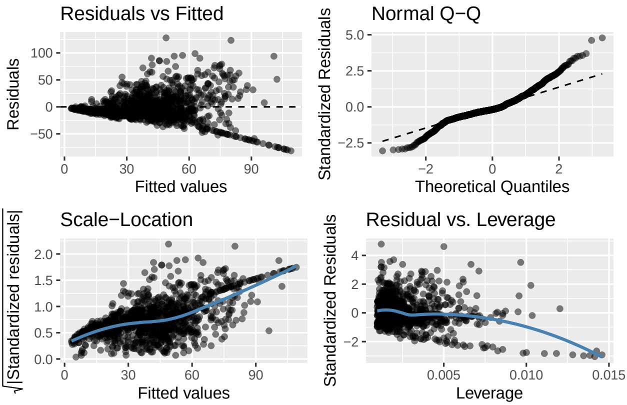
```
tree_lm<-lm(Carbon_Sequestration_lb ~Crown_Width_EW + Tree_Height, data=my_pdxTrees)
summary(tree_lm)
```

```
##
## Call:
## lm(formula = Carbon_Sequestration_lb ~ Crown_Width_EW + Tree_Height,
##     data = my_pdxTrees)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -81.46 -14.03  -4.92  11.51 127.87
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -1.06948    2.02087  -0.529    0.597
## Crown_Width_EW  0.79289    0.04624  17.146  < 2e-16 ***
## Tree_Height     0.12897    0.02820   4.573 5.38e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26.74 on 1031 degrees of freedom
##   (5 observations deleted due to missingness)
## Multiple R-squared:  0.3699, Adjusted R-squared:  0.3687
## F-statistic: 302.7 on 2 and 1031 DF,  p-value: < 2.2e-16
```

Diagnostic Plots

```
library(gglm)
gglm(tree_lm)
```



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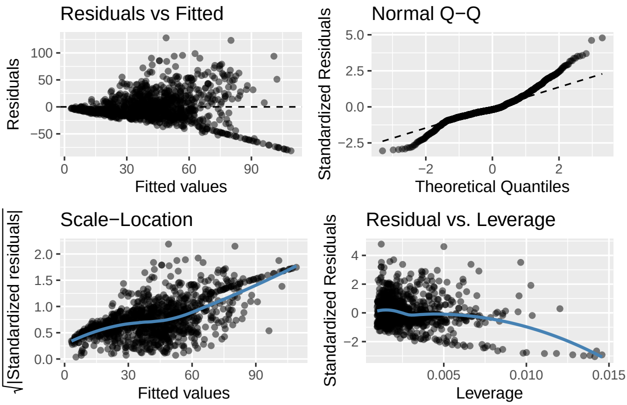
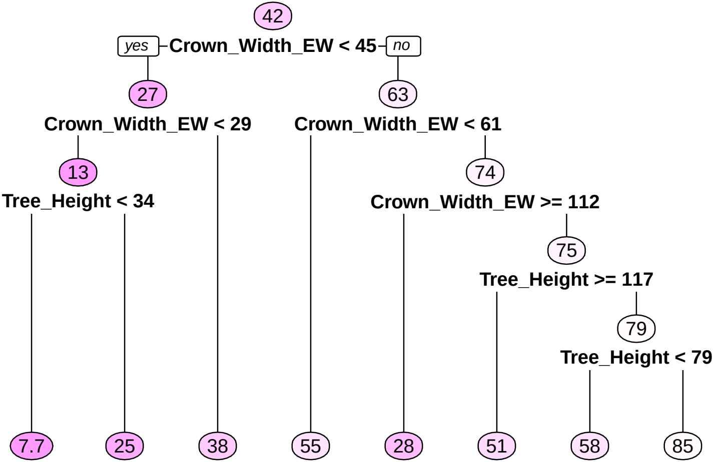
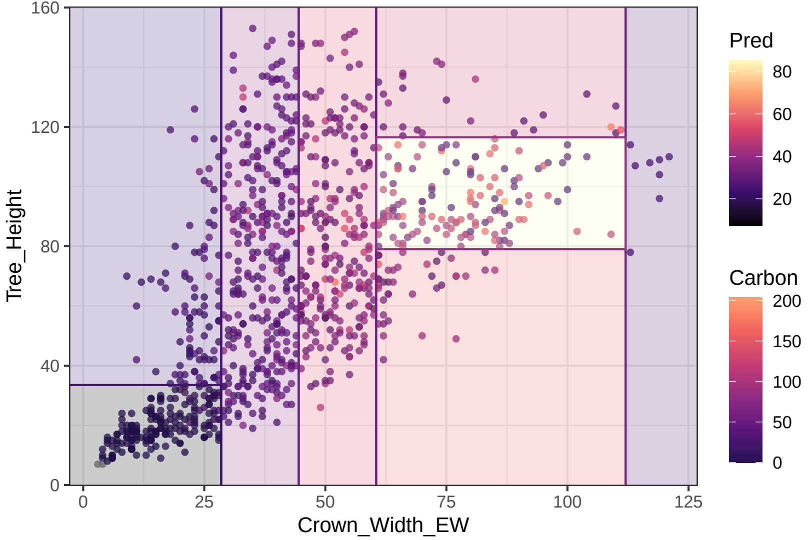


- Concerns?

Regression Tree



Another Visualization



Interpretation

- Crown_Width_EW is the most important factor contributing to Carbon_Sequestration_lb
- After accounting for width, Tree_Height has some impact on Carbon_Sequestration_lb
- Given a narrow tree, shorter trees tend to have lower Carbon_Sequestration_lb
- Given wide tree, moderately tall trees have largest Carbon_Sequestration_lb

Tree Accuracy

Let's create a test set consisting of parks further from Reed:

```
my_pdxTrees_test <- get_pdxTrees_parks(park = c("Mt Scott Park", "Glenwood Park"))
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$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

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Why did the tree model outperform the linear model?

Nevertheless, what are some downsides to the tree model?

Extra Practice

The `mtcars` dataset gives the `mpg`, `hp`, and `wt` for 32 car models.

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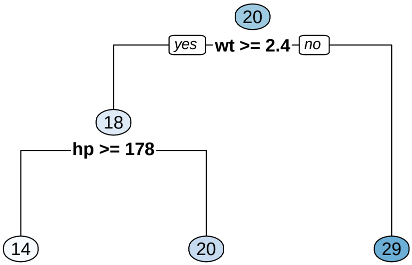
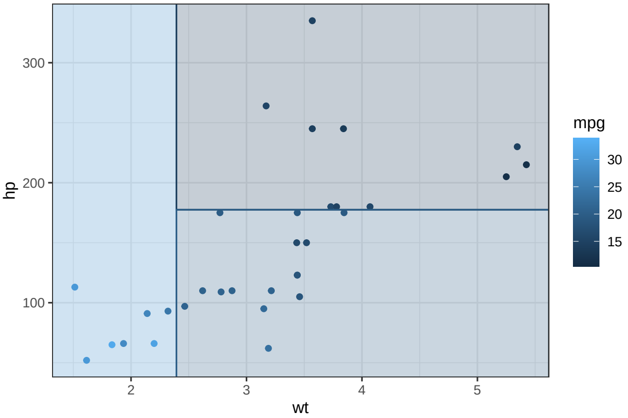
In small groups, draw the predictor space corresponding to the following tree, predicting `mpg` based on `wt` and `hp`.



What would you expect the signs of the corresponding regression slopes to be?

Results




```
##                Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) 37.22727012 1.59878754 23.284689 2.565459e-20
## hp          -0.03177295 0.00902971 -3.518712 1.451229e-03
## wt          -3.87783074 0.63273349 -6.128695 1.119647e-06
```

Section 3

Pruning

The general tree algorithm

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Consider the RSS of a **big** tree. How might training and test RSS compare?

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- But this search is actually even more computationally expensive than best subset!
- So we instead restrict our attention to those subtrees most likely to improve RSS

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1. Choose the tree with smallest MSE.
2. Choose the *smallest* tree with MSE within 1 standard deviation of smallest MSE